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A theoretical drought classification method for the multivariate drought index based on distribution properties of standardized drought indices

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ABSTRACT

Drought indices have been commonly used to characterize different properties of drought and the need to combine multiple drought indices for accurate drought monitoring has been well recognized. Based on linear combinations of multiple drought indices, a variety of multivariate drought indices have recently been developed for comprehensive drought monitoring to integrate drought information from various sources. For operational drought management, it is generally required to determine thresholds of drought severity for drought classification to trigger a mitigation response during a drought event to aid stakeholders and policy makers in decision making. Though the classification of drought categories based on the univariate drought indices has been well studied, drought classification method for the multivariate drought index has been less explored mainly due to the lack of information about its distribution property. In this study, a theoretical drought classification method is proposed for the multivariate drought index, based on a linear combination of multiple indices. Based on the distribution property of the standardized drought index, a theoretical distribution of the linear combined index (LDI) is derived, which can be used for classifying drought with the percentile approach. Application of the proposed method for drought classification of LDI, based on standardized precipitation index (SPI), standardized soil moisture index (SSI), and standardized runoff index (SRI) is illustrated with climate division data from California, United States. Results from comparison with the empirical methods show a satisfactory performance of the proposed method for drought classification.

1. Introduction

The devastating effects of drought and the potential increase in frequency and severity due to climate change have led to extensive studies for better understanding, monitoring and prediction of drought. Drought indices are the key components for monitoring drought, based on meteorological or hydrological variables. A large number of individual drought indicators, such as Standardized Precipitation Index (SPI) (McKee et al., 1993) that is commonly used for meteorological drought monitoring, have been developed in the past few decades for characterizing different aspects of drought across space and time (Ellis et al., 2010; Hao and Singh, 2013; Mishra and Singh, 2010). Many of these drought indices are generally based on the anomaly, percentile (e.g., soil moisture percentile, SMP), or the standardized indices (e.g., SPI) of various hydroclimatic variables. The derivation of SPI applies to a large number of drought indices, such as the Standardized Precipitation Evapotranspiration Index (SPEI), Standardized Soil Moisture Index (SSI), Standardized Runoff Index (SRI) (Hao et al., 2014; Mo, 2011; Shukla and Wood, 2008; Vicente-Serrano et al., 2010), based on the distribution of various hydro-climatic variables. A variety of probability distributions (e.g., gamma, generalized extreme value (GEV), generalized logistic distribution, and beta distributions) have been used to fit monthly observations of different hydroclimatic variables for the computation of drought indices (Guttman, 1999;

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McKee et al., 1993; Sheffield et al., 2004; Stagge et al., 2015; Vicente-Serrano et al., 2010).

Meanwhile, a suite of multivariate drought indices (MDI) or products to integrate drought information from multiple sources have been developed in the past decades (Hao and Singh, 2015; Kao and Govindaraju, 2010; Vicente-Serrano et al., 2010; Ziese et al., 2014), such as the U.S. Drought Monitor (USDM) (Svoboda et al., 2002), Aggregate Drought Index (ADI) (Keyantash and Dracup, 2004), and Multivariate Standardized Drought Index (MSDI) (Hao et al., 2014). For example, the linearly combined drought index (LDI) is among the commonly used multivariate drought indices for drought monitoring and combines different drought indices in a linear manner with associated weights to each index (Mo and Lettenmaier, 2014; Xia et al., 2014a). These multivariate indices have been compared with other commonly used drought indices to assess their performance for drought monitoring due to the lack of the ground truth of drought observations. However, since multivariate drought indices are developed with different methods and may not be directly comparable with univariate indices (e.g., many indices are not developed based on a percentile approach or standardized in the same way), it is important to accurately process and evaluate these indices before implementing them for operational drought management.

Drought management of many regional drought plans requires the determination of a drought threshold to trigger a response (e.g., limiting water use) during the drought event (Quiring, 2009; Steinemann, 2003). Based on a suitable definition of the threshold of drought severity, drought condition can be classified into several drought categories to aid stakeholders and policy makers to implement drought management measures. The USDM has been widely used for various applications for both drought research and application purposes, which classifies the drought into five major categories, including abnormally dry (D0), moderate drought (D1), severe drought (D2), extreme drought (D3), and exceptional drought (D4), from the least intense to the most intense (Svoboda et al., 2002). The classification is based on a percentile approach (e.g., the D1 drought condition corresponds to the 20th percentile) and has been used in several studies to depict the drought condition in the drought category form (Goodrich and Ellis, 2006; Steinemann, 2003; Steinemann et al., 2015). For example, Mo and Lettenmaier (2014) developed the grand mean drought index to combine SPI, SMP and SRI with equal weight, which is then remapped into a uniform distribution that enables the classification of drought severity into different drought categories based on the percentile approach. Other methods have also been developed for drought classification to aid drought management. For example, Mallya et al. (2014) proposed a probabilistic drought classification method based on SPI using a gamma mixture model. However, most of the previous studies are based on the univariate drought index and the drought classification for the multivariate drought index has been less explored, which is even more challenging, since the distribution forms of these drought indices are not explicit or clear.

The objective of this study therefore is to develop a theoretical method for objective drought classification of the multivariate drought index with focus on the linearly combined drought indicator (LDI). By deriving the theoretical distribution of LDI, the proposed method enables the classification of drought severity of LDI into drought categories based on the percentile approach. The proposed method is tested with climate division data from California in the United States and results show the validity of the proposed method for drought classification. This paper proceeds as follows. The method to derive the distribution of multivariate drought indices is introduced in Section 2. The case study to illustrate the application of the proposed drought classification method is presented in Section 3, followed by conclusions and discussion in Sections 4 and 5.

2. Method

2.1. Normal quantile transformation

The normal quantile transformation (NQT) (or normal score, inverse normal) has been commonly used to transform the variable of interest to the Gaussian variable in order to make it more treatable with statistical models (Bogner et al., 2012; Krzysztofowicz, 1997; Montanari and Brath, 2004). The general steps in the NQT include the computation of cumulative probabilities of observations and transformation into normal realizations by applying the standard normal (or Gaussian) distribution. For a random variable Z, the cumulative distribution function (CDF) $F(Z)$ can be expressed as

$$F(Z) = P(Z \le z) \tag{1}$$

The normal quantile transformation of Z (denoted as NQT(Z)) takes the form (Bogner et al., 2012; Krzysztofowicz, 1997; Montanari and Brath, 2004)

$$NQT(Z) = N^{-1}[F(Z)] \tag{2}$$

where N is the standard normal distribution with zero mean and unit standard deviation. This transformation ensures that the distribution of NQT(Z) is Gaussian, regardless of the original distribution form of Z, which fulfills the underlying normal distribution assumption that is intrinsic to many statistical models (Kelly and Krzysztofowicz, 1997; Hao and Singh, 2016).

The concept of deriving SPI has been used to derive the standardized drought index (SDI) based on other hydroclimatic variables, such as soil moisture, runoff, snowmelt, ground water and precipitation minus potential evapotranspiration, for drought characterizations (Hao and Singh, 2015). Due to the standardized nature, SPI (and SDI) values for different time scales can be compared at different locations and seasons, which is a desirable property of drought indicators for drought assessment. With the introduction of NQT in Eq. (2), it becomes clear that the computation of SPI essentially adopts the NQT procedure. Thus, the SDI, such as SPI, SSI, SRI and SPEI, possesses the properties of the NQT variables.

2.2. Distribution of linear combination of normal random variables

Suppose $\mathbf{X}$ is a random vector ($n \times 1$) of the multivariate normal distribution (MVN). Let a random variable Y be composed of $\mathbf{X}$ with the weight $b^{\mathrm{T}} = [b_1, b_2, ..., b_n]$:

$$Y = b^T X \tag{3}$$

Then, the random variable Y is a univariate normal random variable with mean u_y and variance $\sigma^2{}_y$ as (Wilks, 2011)

$$u_y = b^T u_x \quad \sigma^2{}_y = b^T \sum_x b \tag{4}$$

where u_x and Σ_x are the mean and covariance matrix of the random vector $\mathbf{X}$. Eq. (4) states that the linear combination of the MVN vector X is also normal.

Since each SDI is essentially the NQT variable, which is a standard normal random variable, an assumption is made here that the joint distribution of the SDI is a multivariate normal distribution (note that a vector of normally distributed random variables does not imply that the vector has a joint normal distribution). Based on this assumption, the linear combination of SDIs is also normally distributed with mean and variance expressed in Eq. (4).

2.3. Distribution of linearly combined drought index

As a general form, the LDI can be expressed as

$$LDI = \alpha_1 SDI_1 + \alpha_2 SDI_2 + ... + \alpha_n SDI_n \tag{5}$$

where SDIs represent various forms of the standardized drought index, such as SPI, SSI, SRI, and SPEI; and $\alpha_1,...,\alpha_n$ are the weights. When drought indices are computed in a different way from SDI, such as PDSI, they can be transferred to the SDI by fitting a distribution followed by the inverse of the standard normal distribution (Goodrich and Ellis, 2006; Quiring, 2009), which essentially is the NQT procedure.

The main purpose is to explore the distribution property of LDI to facilitate drought classification based on the percentile approach. Since SDI in Eq. (5) is essentially normally distributed, the distribution of LDI can then be derived from the distribution property based on the assumption of multivariate normal distribution, as shown in Section 2.2. From Eq. (4), LDI is a normal random variable with mean u_{LDI} and variance $\sigma^2{}_{\text{LDI}}$. Specifically, by denoting $\boldsymbol{\alpha}^{\mathbf{T}}=[\alpha_1, \alpha_2,...,\alpha_n]$, the mean of LDI can be expressed as

$$u_{LDI} = \alpha_1 u_1 + \alpha_2 u_2 + \ldots + \alpha_n u_n \tag{6}$$

where u_1,u_2,...u_n denoted the mean values of SDIs which are generally close to 0.

The covariance matrix of LDI in Eq. (5) can be expressed as

$$\Sigma_{LDI} = \begin{pmatrix} \text{cov}(SDI_1, SDI_1) & \text{cov}(SDI_1, SDI_2) & \ldots & \text{cov}(SDI_1, SDI_n) \\ \text{cov}(SDI_2, SDI_1) & \text{cov}(SDI_2, SDI_2) & \cdots & \text{cov}(SDI_2, SDI_n) \\ \ldots & \ldots & \ldots & \ldots \\ \text{cov}(SDI_n, SDI_1) & \text{cov}(SDI_n, SDI_2) & \cdots & \text{cov}(SDI_n, SDI_n) \end{pmatrix} \tag{7}$$

Thus, one can obtain the distribution function of LDI from Eq. (4), which can be expressed as

$$LDI \sim N(u_{LDI}, \alpha^T \Sigma_{LDI} \alpha) \tag{8}$$

where α^{T} denotes the weights associated with different SDIs that can be determined empirically or through optimization approaches (Hao and Singh, 2015; Mo and Lettenmaier, 2014; Xia et al., 2014b).

2.4. Drought classification

Since the distribution property of LDI is known from Eq. (8), the drought classification can be achieved based on the percentile recommended by the USDM for the drought category D0 (20 to 30), D1 (10 to 20), D2 (5 to 10), D3 (2 to 5), and D4 ($\leq$ 2) or threshold of the index. Specifically, for SPI (or SDI), these percentiles correspond to the threshold values T = [−0.5, −0.8, −1.3, −1.6, and −2](Svoboda et al., 2002). For LDI, the threshold of index value for each drought category can be obtained from the theoretical distribution in Eq. (8), based on the specified percentile. Specifically, for a specified percentile p_0 (e.g., $p_0 = 2\%$ corresponds to the threshold separating D3 and D4 drought categories), the corresponding quantile x_0 can be expressed as

$$x_0 = N^{-1}(p_0; u_{LDI}, \sigma^2{}_{LDI}) \tag{9}$$

where N^{-1} is the inverse normal distribution function.

Alongside the proposed theoretical method for drought classification of LDI, an empirical method for the classification of multivariate drought indices can be developed by empirically estimating the distribution (and the percentile) of LDI. A suite of plotting position formulas, such as Weibull, Gringorten, Gumbel, Harris, Hazen, Beard, California and others, can be used for the empirical probability estimation (Cook, 2011; Fuglem et al., 2013; Gringorten, 1963; Makkonen, 2006), especially when the sample size is relatively large. The Weibull plotting position formula has been commonly used to empirically estimate probability and was also used to estimate the distribution of LDI in this study. Specifically, the Weibull plotting position formula is expressed as

$$P = \frac{i}{n+1} \tag{10}$$

where P is the probability; n is the length of observed data and i is the rank of the observed values.

The drought classification can then be achieved through the empirical distribution of LDI (or other indices) based on the percentile recommended by the USDM and the standardization can also be obtained through the NQT in Eq. (2). In this study, the empirical drought classification was used as a benchmark for comparison of theoretical drought classification. The advantage of empirical method is that it does not rely on the assumed parametric distribution form.

3. Case study

3.1. Data

Monthly precipitation, soil moisture and surface runoff data for the period 1932–2011 from climate divisions in California were used for the case study to illustrate the application of the proposed method. The locations of different climate divisions are shown in Fig. 1. A one-layer leaky bucket hydrological model (Huang et al., 1996; Van den Dool et al., 2003) was used to calculate soil moisture, evaporation and runoff with observed monthly precipitation and temperature over 344 climate divisions from National Climatic Data Center (NCDC) as forcing variables (assuming a soil column depth of 1600 mm). These climate division data were obtained from the Climate Prediction Center (CPC), National Oceanic and Atmospheric Administration (NOAA) (ftp://ftp.cpc.ncep.noaa.gov/wd51yf/us).

The monthly data were used for the computation of different drought indices, including 6-month SPI, 1-month SSI, and 3-month SRI representing meteorological, agricultural and hydrological droughts, respectively. In this study, the Weibull plotting position formula was used to compute these drought indices, which were then used to obtain the linearly combined drought index to evaluate the performance for drought classification. The proposed method was also tested based on data from other climate divisions and similar results were found (not shown).

3.2. Comparison of univariate and multivariate drought indices

Three drought indices, SPI, SSI and SRI, were computed based on monthly data from 1932 to 2011 (80 years) for Climate Divisions 5 (denoted as CD 5) and Climate Divisions 7 (denoted as CD 7), which are San Joaquin Drainage and Southeast Desert Basin divisions, in California. A plot of these indices for the period 1998–2011 is shown in Fig. 2. Generally, all three drought indices showed historical drought conditions in California, such as the 2002 drought and the 2007–2009 drought (with indices lower than -0.8). However, these three drought indices performed differently in depicting the onset, severity, and end of drought condition, which have been highlighted in a few studies (Hao et al., 2014; Mo, 2011; Shukla and Wood, 2008). The differences in characterizing drought conditions are understandable, since generally drought originates from precipitation deficit (meteorological drought), which then leads to the depletion of soil moisture (or agricultural drought) with the onset generally lagging that of meteorological drought, and the deficiency of streamflow or groundwater resulting in the hydrological drought (Entekhabi et al., 1996; Hao and Singh, 2015; Heim, 2002). For example, for the drought condition during 2008 in CD 5, meteorological drought (SPI) generally precedes agricultural drought (SSI) and hydrological drought (SRI). This also shows that an individual index is generally not sufficient to characterize complicated drought conditions for all seasons and regions, and thus accurate drought monitoring requires the integration of multiple drought related variables or indicators.

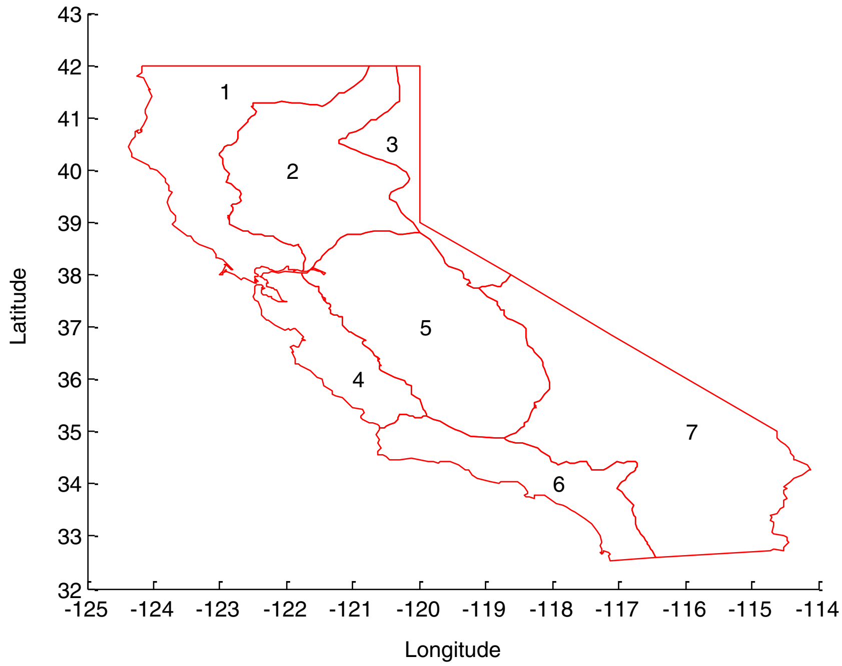

Fig. 1. Locations of climate divisions in California, USA.

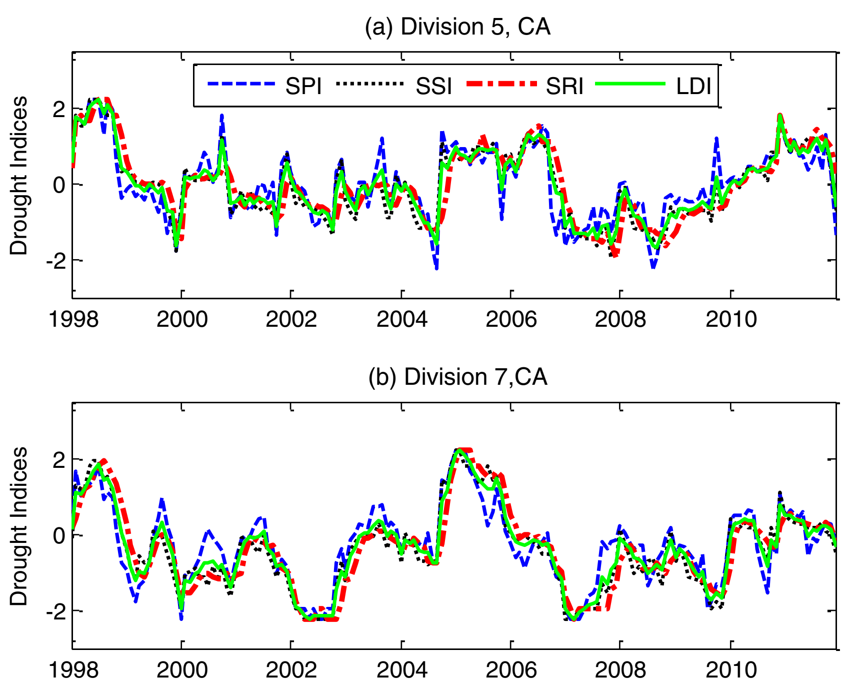

Fig. 2. Plot of monthly drought indices SPI, SSI, SRI, and LDI (between January 1998 and December 2011) for climate divisions 5 and 7 in California (CA), USA.

In this study, a linear combination of three drought indices SPI, SSI and SRI was used to integrate drought information from meteorological, agricultural and hydrological droughts to illustrate the application of the proposed drought classification method. Specifically, the linearly combined drought index in Eq. (5) for this case was defined with $SDI_1 = SPI$, $SDI_2 = SSI$ and $SDI_3 = SRI$. Following Mo and Lettenmaier (2014), here weights of the three drought indices were assigned to be equal ($\alpha_1 = \alpha_2 = \alpha_3 = 1/3$). The LDI values for the period 1998–2011in CD 5 and CD 7 in California are also shown in Fig. 2, in which the LDI clearly shows historical drought events in California, such as the 2007–2009. For example, for October 2008 in CD 5, values of SPI, SSI and SRI were −1.01, −1.44, and −1.36, respectively, and LDI_0 was −1.27. Basically, LDI consolidates drought information with the average of the three drought indices. Notice that different drought indices convey different aspects of drought condition and thus LDI is not meant to be superior to the univariate drought index. In the operational drought management, LDI can be used along with other drought indices to characterize drought.

3.3. Drought classification

Here the LDIs for October in CD 5 and July in CD 7 in California were used for illustrating the drought classification results. The

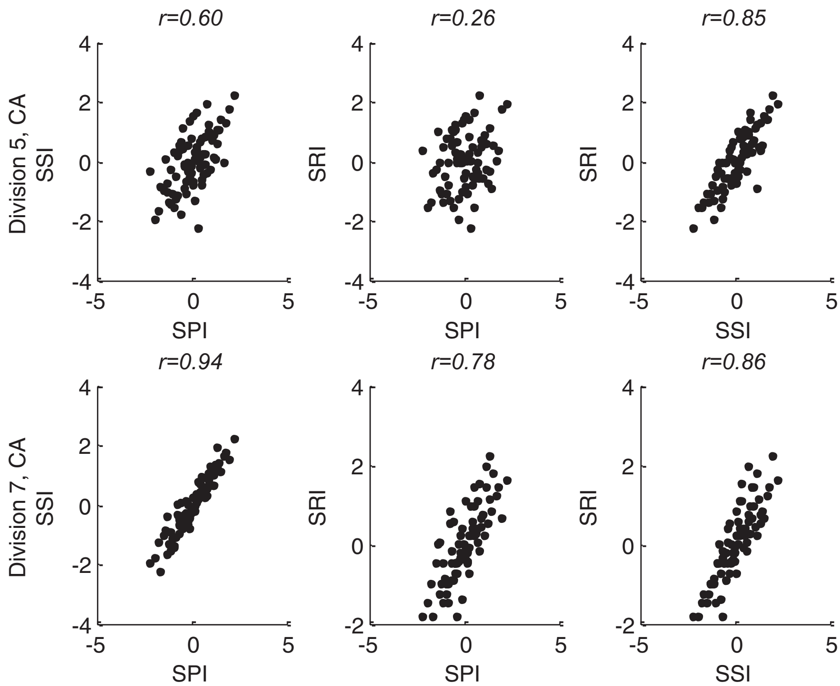

Fig. 3. Scatter plots and correlation coefficients (r) of three drought indices SPI, SSI, and SRI for October in climate division 5 and July in climate division 7 in California, USA.

scatter plots of pairs of drought indices SPI, SSI and SRI for July and October (sample size $n = 80$) in these two climate divisions are shown in Fig. 3, along with the Pearson correlation coefficient (r) to measure the stochastic dependence between two drought indices. The SPI-SSI and SSI-SRI were highly correlated in the specific month of the two climate divisions. However, the correlation between SPI-SRI for October in CD 5 was rather weak, while that for July in CD 7 was relatively strong. For drought indices of October in CD 5 (July in CD 7) in California, the mean and standard deviation of LDI estimated from Eqs. (6) and (7) were 0 (0) and 0.81 (0.91). These results indicate that the variance of LDI depended on the stochastic dependence of drought indices SPI, SSI and SRI and different degrees of correlations (or covariance) would result in different values of the variance of LDI (may differ from the unity). Thus, the distribution of LDI may not be standard normal and Eq. (9) can be used to aid the classification in this case.

The theoretical cumulative distribution function (CDF) of LDI (sample size $n = 80$) for the two months is shown in Fig. 4(a) and (b), respectively, along with the empirical distribution based on the Weibull plotting position formula. It can be seen that the theoretical CDF was close to the empirical CDF, implying that the theoretical distribution in Eq. (8) was generally valid. The Kolmogorov–Smirnov (K–S) test (Bloomfield and Marchant, 2013) was used for the goodness-of-fit of the theoretical distribution with a sample size of 80 and results indicated that the null hypothesis (i.e., data come from the normal distribution) could not be rejected at the 5% significance level, which confirmed the validity of the theoretical distribution. We also assessed the validity of the distribution of LDI in all climate divisions in California for different months and record length ($n = 30$ and 50) based on the Kolmogorov–Smirnov (K–S) test and it was found that the theoretical distribution was valid for almost all cases.

After the validation of the distribution property of LDI, the drought classification for each month can then be achieved by computing the associated threshold of drought indices based on the percentiles suggested by USDM from Eq. (9). For October in CD 5, for example, the threshold value corresponding to the 5th percentile was estimated as −1.33, which differed significantly from the threshold for the SPI recommended by USDM (−1.6), since the distribution of LDI was not standard normal. For July in CD 7, due to the relative closeness of the standard deviation (0.91) to unity, the threshold values for drought classification were relatively similar to those recommended by USDM for the SPI.

To evaluate the overall performance of the proposed method for drought classification, the percentages of different drought categories for the whole period from 1932 to 2011 (80 years) for all climate divisions (totally 7) in California were obtained, as shown in Fig. 5 with boxplots. The central mark of the box is the median of drought percentages with edges of the box representing 25th and 75th percentiles and whiskers of the box representing the maximum and minimum values of the percentages. Note that the number of drought categories estimated from the empirical method, which depended on the rank of the data, for different months (and climate divisions) was the same and thus the percentages of drought categories from the empirical method were generally the same for all climate divisions. Overall, the drought percentage from the empirical method fell within the boxplot, indicating a satisfactory performance of the drought classification from the proposed method. For example, the median of the drought percentage for the D3 drought category from the proposed method was 2.8%, which was close to the value of 2.5% estimated from the empirical method. These results showed that the proposed method based on the theoretical distribution of linearly combined drought indices through comparison with the empirical method performed relatively well for drought classification.

4. Discussions

The theoretical method mainly applies to the multivariate drought index based on the linear combination while the empirical method can be used for any multivariate drought index based on empirical estimation of the percentile (or probability). Several multivariate drought indices, such as MSDI and ADI, have been developed in recent years with various methods (but not the

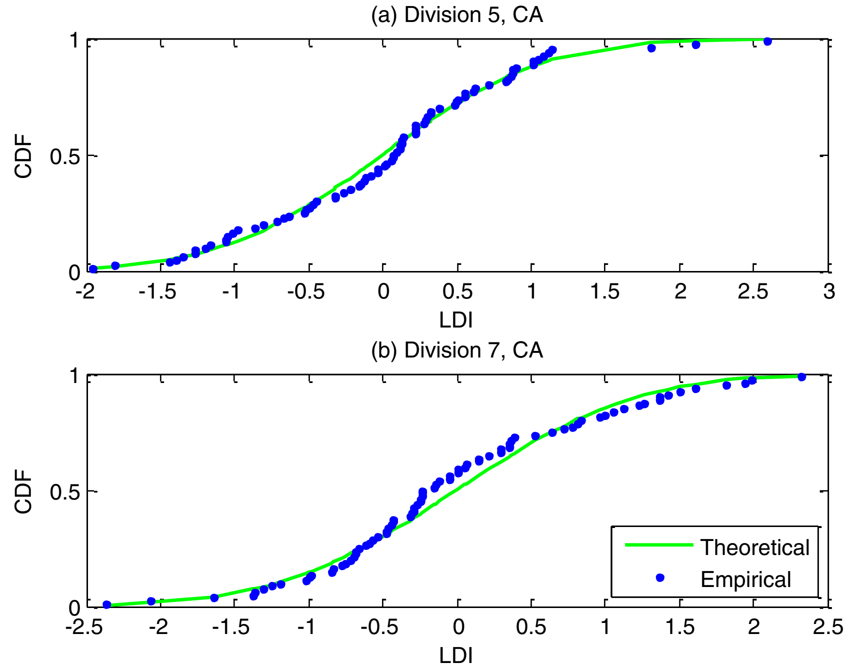

Fig. 4. Theoretical and empirical distributions of LDI for October in climate division 5 and July in climate division 7 in California, USA.

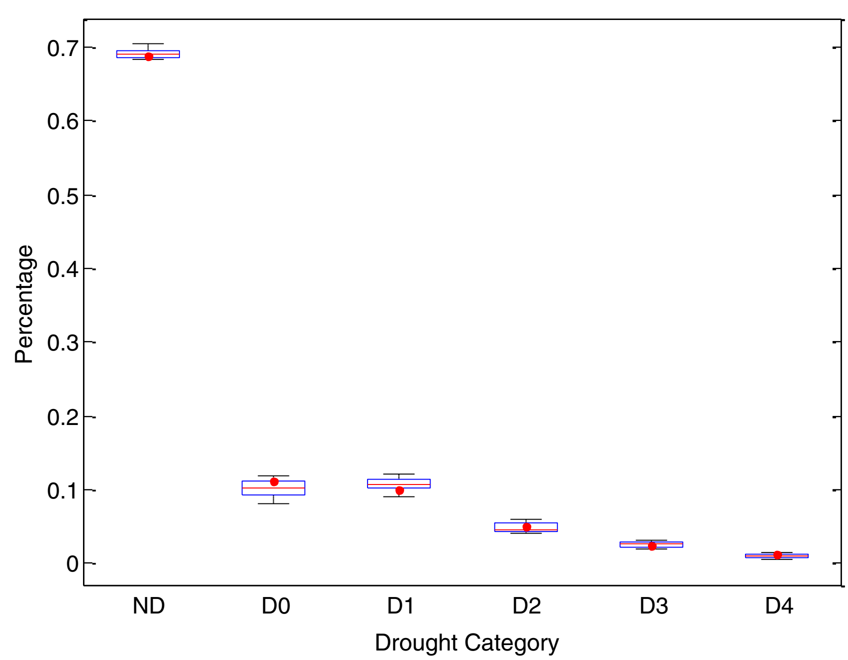

Fig. 5. Comparison of the percentages of different drought categories from the theoretical method (shown with boxplot) and empirical method (shown with point) for all climate divisions in California, USA.

linear combination) and the distribution property is generally unclear. In this case, the empirical method can be used to obtain the percentile (or standardization) of the index to facilitate drought classification. The MSDI, which has been proposed to incorporate drought information from multiple sources based on the joint distribution (Hao et al., 2014), was used in this section for illustrative purposes. The trivariate MSDI based on three random variables X_1, X_2 and X_3 can be expressed as

$$MSDI = \phi^{-1}[P(X_1 \le x_1, X_2 \le x_2, X_3 \le x_3)] \tag{11}$$

where φ is the standard normal distribution; and $P(X_1 \le x_1, X_2 \le x_2$ and $X_3 \le x_3)$ is the joint percentile (or probability).

Though MSDI is standardized with the normal distribution, it is generally not normally distributed since the distribution of the joint percentile in Eq. (11) is not uniform within [0 1]. To illustrate this point, the MSDI based on three indices SPI, SSI and SRI for two months (August and October) from 1932 to 2011 for CD7 in California is shown in Fig. 6. It can be seen that generally more values of MSDI falls below 0. For example, the percentage of the MSDI values below 0 and -0.5 for October is 75% and 54%, respectively. In addition, none of the two samples passed the Kolmogorov–Smirnov normality test (sample size $n = 80$). The transformed MSDI (termed as $MSDI_t$) based on NQT with the empirical method for the two months is also shown in Fig. 6. Due to NQT, the distribution of

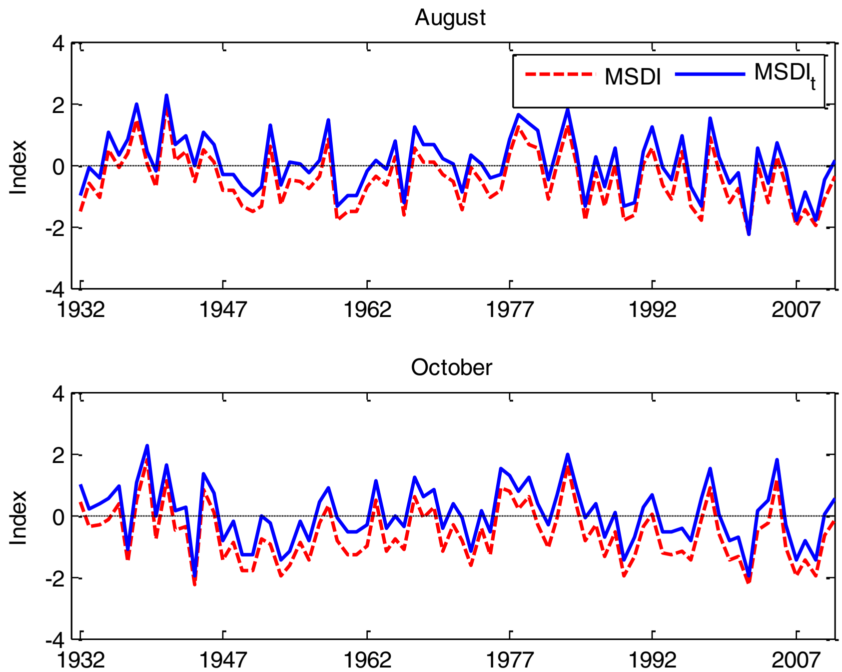

Fig. 6. Comparison of the trivariate MSDI and its transformation (MSDI$_t$) based on the empirical distribution for August and October for the period 1932–2011 (sample size $n = 80$) for climate division 7 in California, USA.

MSDI$_t$ for the two months passed the Kolmogorov–Smirnov (K–S) test (sample size $n = 80$). The difference between MSDI and MSDI$_t$ would result in the difference in drought classification. For example, the percentage of MSDI$_t$ values below 0 and −0.5 for October was 50% and 30%, respectively. As such, the MSDI$_t$ can be employed for the drought classification and monitoring in a similar way as other standardized drought indices such as SPI and SPEI. Detailed comparison of the performance of MSDI$_t$ for drought monitoring is beyond the scope of this study and will be carried out in the future.

5. Conclusions

From an application standpoint, it is important to define a threshold for drought indices to trigger a response for drought management. In this study, a statistical method for the objective drought classification of LDI is proposed by deriving its theoretical distribution function to classify drought categories based on a percentile approach. An empirical method is also employed for the drought classification to assess the performance of the proposed method. Based on three drought indices, including SPI, SSI, and SRI, representing meteorological, agricultural and hydrological drought in climate divisions in California in the United States, the proposed method is evaluated and results show a satisfactory performance for drought classification of the LDI.

The proposed model is also capable of incorporating other drought indices that are based on the same standardization method as SPI, such as SPEI (Vicente-Serrano et al., 2010). Moreover, drought indices, such as PDSI, which are not based on the standardization, can also be embedded in this framework through the normal quantile transformation (NQT). The advantage of the proposed method over the empirical method is that it can be used for the statistical extrapolation beyond historical observations, while the empirical method generally falls short in this regard. However, the empirical method can be applied to any multivariate drought index to facilitate drought classification with a percentile approach, for which a relatively large sample size is generally required for the accurate estimation of distribution functions in this case. In the past decade, various univariate and multivariate drought indices have been assessed and compared with each other (and USDM) for drought monitoring across space and time. Due to differences in univariate and multivariate indices, it is recommended to evaluate the multivariate drought indices by transforming them into categories or percentiles with the proposed theoretical or empirical method to ensure the statistical consistency across space and time for operational drought management.

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